

**Al-Taftāzānī on the Liar Paradox**

Truth, Goodness, Liar Cycles, and the “Fallacy of the Irrational Root (*al-Jadhr al-aṣamm*)”<sup>\*</sup>

*David Sanson and Ahmed Alwishah*

1. INTRODUCTION

Sa‘ad al-Dīn Mas‘ūd ibn ‘Umar ibn ‘Abd Allāh al-Taftāzānī (1322–1390) was a Persian polymath who wrote on a wide range of subjects—grammar, logic, theology (*kalām*), jurisprudence (*fiqh*), and literature. Here we translate and discuss his remarks on the Liar Paradox, in which he (a) presents the first example of a Liar Cycle or Deferred Liar in the tradition, (b) gives the paradox a puzzling name—the fallacy of the “irrational root” (*al-jadhr al-aṣamm*)—which became the standard name for the paradox in the tradition, and (c) suggests a connection between the paradox and what it tells us about the nature of

---

<sup>\*</sup>Both authors would like to thank Calvin Normore, Terry Parsons, Roshdi Rashed, and an anonymous referee. The first author would also like to thank Andy Arlig, Jeff Behrends, Daniel Breyer, Sunil Chebolu, Harry Deutsch, Keota Fields, and Todd Stewart for helpful discussion, comments, and feedback. Material from this paper was presented at the 2015 NEH Summer Institute, “Between Medieval and Early Modern Philosophy from 1300-1700,” directed by Robert Pasnau, at the University of Colorado, Boulder. Any views, findings, conclusions, or recommendations expressed in this paper do not necessarily represent those of the National Endowment for the Humanities.

truth and falsehood, and related puzzles concerning reason and the nature of goodness and badness.

Al-Taftāzānī's discussion occupies a unique position in the tradition. The century prior had yielded a flurry of attempts to solve the paradox, centered around the Maragha School, with the work of Athīr al-Dīn al-Abharī (1200-1265), Naṣīr al-Dīn al-Ṭūsī (1201-1274), Kātibī al-Quzwīnī (d.1276), Sa'ad Ibn Mansūr Ibn Kammūna (d.1284), and Shams al-Dīn Muḥammad al-Samarqandī (1240-1304).<sup>1</sup> The following century would yield the first treatises dedicated to the problem, centered around the Shīrāz school, with the work of Ṣadr al-Dīn al-Dashtakī (d. 1497) and Jalāl al-Dīn al-Dawwānī (1426-1502).<sup>2</sup>

These later authors seem to give al-Taftāzānī pride of place. They begin with him, and follow him in calling it the fallacy of the irrational root. But they also seem dismissive.

Al-Dawwānī says,

---

<sup>1</sup> For a brief overview of the place of the Maragha School in the history of Arabic logic, see Tony Street, "Arabic and Islamic Philosophy of Language and Logic." In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Winter 2013. For extended discussion of the solutions offered by al-Abharī and al-Ṭūsī, and references to al-Quzwīnī, Ibn Kammūna, and al-Samarqandī, see Alwishah and Sanson, "The Early Arabic Liar: The Liar Paradox in the Islamic World from the Mid-Ninth to the Mid-Thirteenth Centuries CE." *Vivarium*, 47 (2009), 97–127. Also see Reza Pourjavady and Sabine Schmidtke, *A Jewish Philosopher of Baghdad. 'Izz al-Dawla Ibn Kammūna (d. 683/1284) and His Writings* (Leiden and Boston: E.J. Brill, 2006), 41–44.

<sup>2</sup> We are indebted to Ahad Qarāmalekī for editing and collecting most of the texts in the tradition that discuss the Liar, most of which have been collected together in his *12 Treatises on Liar Paradox in Shirāz School* (Tehran: Iranian Institute of Philosophy, 2007).

Since he pointed to its weakness, we leave [his proposed solution] aside to discuss the others, and we should not explore what is in it.<sup>3</sup>

And al-Dashtakī says much the same,

Since the weakness in [his] solution is apparent to ‘the people of understanding’<sup>4</sup> and [he] admits that he has departed from the right path, for the sake of brevity, we leave aside the details.<sup>5</sup>

Even so, it seems that his introduction of Liar Cycles was influential, as they play an important role in shaping al-Dawwānī’s own views about how to solve the paradox.

## 2. THE CONTEXT

The earliest discussions of the Liar in the tradition situate the problem as a putative counterexample to the definition of ‘declarative sentence’ (*khābar*). So, for example, al-Baghdādī’s argues that,

There is no declarative sentence that is both true and false together, except one: namely, the declaration by he who has not lied at all, about himself, that he is a

---

<sup>3</sup> Al-Dawwānī, *al-Muntakhab min ḥāshiyatihi ‘alā al-sharḥ al-jadīd lil-tajrīd*, in Qarāmalekī, *12 Treatises*, 67.

<sup>4</sup> ‘The people of understanding’ (*li-ūlā al-albāb*) is a quote from Qur’ān verse 2, 179.

<sup>5</sup> Al-Dashtakī, *al-Muntakhab min ḥāshiyatihi ‘alā al-sharḥ al-jadīd lil-tajrīd*, in Qarāmalekī, *12 Treatises*, 5.

liar, and this declarative sentence, from him, is false. And a liar who declares that he is a liar says the truth. And therefore this one declarative sentence is true and false, and it has one subject.<sup>6</sup>

Within the Maragha School, and later, the Shīrāz School, the Liar is instead seen as a “difficult fallacy” (*mughālaṭa ṣa‘ba*) which raises questions about the nature of truth and falsehood, and the relationship between a sentence and what it is about.

But al-Taftāzānī’s discussion of the Liar occurs in a very different context. It appears as an aside in the midst of a broader anti-rationalist polemic against the Mu‘tazila and in favor of a broadly Ashā‘ira theology.<sup>7</sup> The goal of the polemic is to establish that goodness and badness must be *shar‘iyān*—that is, grounded in divine command and so knowable only by revelation.

The discussion occurs in the chapter, “No Judgment for the Intellect On Goodness and Badness,” in al-Taftāzānī’s *Sharḥ al-maqāṣid (Commentary on the Aims of [Kalām])*.

That chapter begins by setting up the terms of this broader dispute:

---

<sup>6</sup> Al-Baghdādī, *Uṣūl al-dīn* 13, 217. For further discussion of this passage from al-Baghdādī, see Alwīshah and Sanson, “Early Arabic Liar,” 101.

<sup>7</sup> For an overview of the debate between Mu‘tazila and Ashā‘ira on the problem of good and evil see George Hourani, *Reason and Tradition in Islamic Ethics* (Cambridge: Cambridge University Press, 1985). Qarāmālekī briefly discusses the connection between these debates and al-Taftāzānī’s treatment of the Liar in his introduction to *12 Treatises*.

Goodness and badness are in our view *shar‘iyan*, but for the Mu‘tazila are rational (*‘aqlī*).<sup>8</sup>

To say that goodness and badness are *shar‘iyān* is to say that they are grounded in divine command, and so only knowable by divine revelation. To say that goodness and badness are “rational” is to say that they are knowable by reason, because they are grounded in the natures of the acts, or the acts together with their consequences, or, as al-Taftāzānī often says, because they are “intrinsic” (*dātī*) to the acts and their consequences.<sup>9</sup>

The aim of the chapter, then, is to establish that goodness and badness are not intrinsic in this sense. To this end, al-Taftāzānī presents several arguments, each meant to show that we fall into incoherence if we try to use reason to judge whether or not something is good or bad.

The *Sharḥ al-maqāṣid* is a commentary on Fakhr al-Dīn al-Rāzī’s *al-Maṭālib al-‘āliyyā min al-‘ilm al-ilāhī* (*The Higher Inquiry of the Science of Metaphysics*), and many of al-Taftāzānī’s arguments derived from al-Rāzī.<sup>10</sup> For example, they both argue that, as judged by reason alone, no human act is good or bad, since all human acts are determined by prior causes. But, they say, this is an absurd consequence, and it follows from the assumption that we can use reason alone to judge the goodness and badness of human

---

<sup>8</sup> Al-Taftāzānī, *Sharḥ al-maqāṣid*, 282.

<sup>9</sup> See also al-Taftāzānī, *Tahdhīb al-manṭq wa al-kalām*, 92.

<sup>10</sup> Al-Rāzī, *al-Maṭālib*, 317–358.

acts, and so that assumption should be rejected.<sup>11</sup>

Another of al-Taftāzānī's arguments concerns the goodness and badness of lying,

If the badness of lying is intrinsic, then by necessity there is no case in which this would be undermined. But this is not true, for what if you have to lie to save the prophet from death? Then you must, and thus the lie becomes good.<sup>12</sup>

The same argument was pressed by al-Rāzī, and, before him, by al-Ghazālī.<sup>13</sup> The position that is being attacked—that lying is intrinsically and absolutely bad, and so

---

<sup>11</sup> Al-Taftāzānī, *Sharḥ al-maqāṣid*, 288. See al-Rāzī, *al-Maṭālib*, 323–324, 328–329.

<sup>12</sup> Al-Taftāzānī, *Sharḥ al-maqāṣid*, 285.

<sup>13</sup> See al-Rāzī, *al-Maṭālib*, 336–337. Al-Ghazālī discusses lying to save a prophet or saint in at least two places. The first is at *Mi'yār al-'ilm*:

Another example is when one judges truthfulness to be good because it is found to agree with one's ends in view, desirable in most cases, but forgets that it is bad on the part of someone who is asked to reveal the place of a prophet or a holy man sought by a questioner in order to kill him. One, (on the other hand), may, (in such an instance) believe the falsehood involved in concealing the place of the prophet to be a bad thing because one has found badness and falsehood to be associated in most cases. (Translation taken from Michael Marmura, "Ghazālī on Ethical Premises," *Philosophical Forum* 1 (1969), p. 401.)

He later discusses lying to save a saint, at *al-Qusṭās al-mustaqīm*, 60:

For someone says: "Every lie is bad in itself."

Then we say: "If one saw one of the saints who has hidden himself from an aggressor, then the aggressor asks him about place of the saint, and he conceals the truth [by saying I do not know], then is what he said a lie?"

He says: "Yes."

We say: "Is it, then bad?"

wrong in all cases—was defended by one of the most prominent Mu‘tazila, ‘Abd al-Jabbār al-Qāḍī (d. 1024).<sup>14</sup> Lying to save the prophet is meant to be a counterexample to this claim. One might take the counterexample to show that reason does not judge lying to be absolutely bad, but al-Taftāzānī takes it to show that reason falls into error when it attempts to judge goodness and badness on its own. This suggests that he thinks that reason alone *does* judge that lying is absolutely bad, but *also* judges that lying to save the prophet is good, and so leads us to contradiction.

Al-Taftāzānī considers a possible response to the counterexample,

---

He says “No, rather what is bad is telling the truth which would result in the death of [the saint].”

We say to him: “Look to the balance.”

And then we say: “His statement — in which he concealed the place of [the saint]— is false, and this is a known principle, and this statement is not bad, and this is the second known principle. Therefore not every lie is bad.”

Now reflect: “Is doubt about this conclusion conceivable, after the admission of the two principles?”

(This is our translation, but we benefited from Richard McCarthy’s translation, in his al-Ghazālī, *Just Balance*. (Boston: Twayne Publishers, 1980), 301.)

For discussion of the broader context of these passages, see Hourani, *Reason and Tradition*, 155. For an overview of al-Rāzī and his relation to al-Ghazālī, see Ayman Shihadeh, “From al-Ghazālī to al-Rāzī: 6th/12th Century Developments in Muslim Philosophical Theology.” *Arabic Sciences and Philosophy*, 15 (2005), 141–179.

<sup>14</sup> Al-Jabbār al-Qāḍī’s devotes volume IV *al-Mughnī* to debates on the problem of goodness and badness. For the claim that al-Jabbār’s view is that lying is absolutely wrong and bad in all cases, see Hourani, *Reason and Tradition*, 112–113.

One might object that in this case lying is still bad, but not saving the prophet is worse than lying, and thus one must do the lesser evil. Thus the requisite good is the saving of the prophet, not the lying.<sup>15</sup>

Here the idea seems to be that lying is always bad, but not always wrong, since the badness of lying can be outweighed by other goods. His response is brief,

Given that the lying is the cause and the reason for the requisite saving, it is also required, and thus it becomes good.

Here the claim seems to be that, even if we distinguish these two acts—lying and saving the prophet—each is required in this case, and so both are good: the saving is required, and the lying is the “cause and reason”—i.e., the necessary means—for the saving, and so it is also required. So once again al-Taftāzānī draws the moral that attempting to judge goodness and badness by reason alone leads to incoherence.

### 3. THE TEXT

We turn now to a third argument, again about the goodness or badness of lying. This is the argument that leads to a discussion of the Liar Paradox. This argument does not seem to be derived from al-Rāzī, and seems to be original to al-Taftāzānī. Since it has not been

---

<sup>15</sup> Al-Taftāzānī, *Sharḥ al-maqāṣid*, 285.



translated before, and we will be discussing it throughout the rest of the paper, we begin with a translation of the passage in full.<sup>16</sup>

[1.] If goodness (*ḥasan*) and badness (*qubḥ*) are intrinsic, then this entails the conjunction of two contradictories, as in the case of the one who says, ‘I will lie tomorrow.’ For either he is telling the truth (*ṣidq*), which must be both a good in virtue of his truthfulness and a bad in virtue of entailing lying tomorrow; or he is lying, which must be both a bad in virtue of the lying and a good in virtue of entailing refraining from lying tomorrow. And thus, it is possible to establish that, there is a conjunction of two contradictories in the statement, ‘I will lie tomorrow.’

[2.] For, in virtue of the lying it [i.e., lying tomorrow] is bad and in virtue of entailing the truth of the original sentence it is good. Or, it is [only] good, but then badness cannot be intrinsic to lying. Or, it is [only] bad, but then refraining from lying becomes good, while also entailing the falsity of the original sentence, and so is [also] bad. And the ground of this entailment is the restriction of the ‘tomorrow’ saying in this case. And thus it can be either true or false, and either way, this entails the conjunction of goodness and badness in it. And the ground

---

<sup>16</sup> Al-Taftāzānī, *Sharḥ al-maqāṣid*, 286–287. Note that the paragraph numbers are our own, and that al-Dawwānī, al-Dashtakī, and al-Khafri only quote paragraphs [4] through [9], and do not discuss paragraphs [1]–[3].

for all this is that goodness must be good, and that badness must be bad, and that each good or bad is intrinsic.

[3.] And this fallacy can be set up in such a way that truth and falsehood are gathered in one sentence (*kalām*), and thus goodness and badness are gathered. For if we consider a proposition (*qadiyya*) whose purport is to declare itself not to be true, then truth and falsehood are entailed in it, as when you say that the sentence (*kalām*) I now speak is not true, for its truth entails the non-being of its truth and vice versa.

[4.] And this can be expressed in the form of ‘tomorrow’ and ‘yesterday’ sentences (*kalām*). For if one says that the sentence (*kalām*) I speak tomorrow is not true, or that nothing I speak tomorrow is extrinsically true, and then tomorrow his only statement is, ‘the sentence (*kalām*) I spoke yesterday is true,’ then the truth of either the ‘tomorrow’ sentence or the ‘yesterday’ sentence entails the non-being of the truth in both cases, and vice versa.

[5.] This is a fallacy that has perplexed the minds of the most intelligent and smartest people. For this reason I call it the fallacy of the irrational root (*jadhr al-ašamm*). I have reviewed many claims, and found nothing that quenched my thirst, and I have contemplated it many times, but nothing came to me except the littlest bit of a little bit, namely:

[6.] In the same way that truth and falsehood are conditions for judgment (*hāllan li-l-ḥukm*)—i.e., affirmative and negative properties, required of all propositions—they can be a judgment (*ḥukman*)—i.e., what is judged [in the sense that it] is predicated of something indirectly, as when we say, this is true and that is false.<sup>17</sup> And they are not contradictory to each other unless we consider them as two conditions for one judgment, or two judgments upon one subject, as opposed to if we consider one of them as a condition for judgment and the other as a judgment. For what differentiates the subject (*al-marja'*) is either some explicit difference, as in our saying, “‘the sky is beneath us’ is true (or false),” or something hidden, as in the specific proposition that belongs to this fallacy.

[7.] So, if we suppose it [i.e., the proposition that belongs to this fallacy, ‘the sentence I now speak is not true,’] to be false, then nothing beyond the truth of its contradiction is entailed, that is, our saying of this sentence that it is true, so then truth occurs as a judgment on this specific [proposition], not as a condition for its judgment; rather the condition for its judgment is falsehood, as we are supposing, and truth is the affirmative property which is a condition for the judgment of the opposite [proposition], and a judgment for the original specific [proposition], and thus it is impossible to gather two conditions for one judgment or two judgments

---

<sup>17</sup>‘Judgment’ (*ḥukman*) is the nominalization of the verb, ‘to judge’ (*ḥukm*), and it is ambiguous. It can refer to a judgment as a whole (e.g., ‘the sky is beneath us’) or it can refer to the predicate judged (e.g., ‘beneath us’).

upon one subject.

[8.] And likewise, if we assume it [i.e., the proposition that belongs to this fallacy, ‘the sentence I now speak is not true,’] to be true, then one may deny the contradiction of the truth and falsity which necessitate each other by referring one of them to the judgment of the specified [proposition] and the other to the judgment of its subject.

[9.] However, the correct judgment regarding this proposition is to give up on a solution and admit the incapability of [solving] this paradox.

#### 4. “THE CONJUNCTION OF GOODNESS AND BADNESS”

We begin by reconstructing the argument, in paragraphs [1] and [2], for the conclusion that goodness and badness are not intrinsic. The central claim of the argument is that if I were to say today, “I will lie tomorrow,” then my doing so would be both good and bad, which is impossible; and so once again the attempt to judge goodness and badness by reason alone leads to incoherence.

For the sake of concreteness, suppose today is  $t_1$  and tomorrow is  $t_2$ , and that what I say tomorrow is “I am not a believer:”

$t_1$ : I say, “I will lie tomorrow”

$t_2$ : I say, ‘I am not a believer.’

At  $t_1$ , either I told the truth or I lied. (Following al-Taftāzānī's lead, we here ignore any distinction between lying and speaking falsely). So suppose I told the truth at  $t_1$ , and so lied at  $t_2$ . Reason tells us that,

T. Truth-telling is always good,

and,

F. Lying is always bad.

So, at  $t_1$ , I did something good, and at  $t_2$ , I did something bad.

It remains to be shown that what I did at  $t_1$  was bad. For this, we need a bridge principle, to get us from the badness of what I did at  $t_2$  to the badness of what I did at  $t_1$ . Al-Taftāzānī says that what I did at  $t_1$  is “bad in virtue of entailing lying tomorrow.” This suggests the principle,

EBB. If doing A entails doing B and doing B is bad, then doing A is bad.

By applying (EBB), we can infer the badness of what I did at  $t_1$  from the badness of what I did at  $t_2$ . So what I did at  $t_1$  was both good and bad.

If we suppose instead that I lied at  $t_1$ , then we can immediately infer from (F) that what I did at  $t_1$  is bad and, from (T), that what I did at  $t_2$  is good. Again, we need a bridge principle, this time to get us from the goodness of what I did at  $t_2$  to the goodness of what I did at  $t_1$ . Again, al-Taftāzānī appeals to entailment, claiming that what I did at  $t_1$  “must

be both a bad in virtue of the lying and a good in virtue of entailing refraining from lying tomorrow.” This suggests the principle,

EGG. If doing A entails doing B and doing B is good, then doing A is good.

Using this principle, we can again infer that what I did at  $t_1$  is both good and bad. So when I said “I will lie tomorrow,” I did something both good and bad, whether I was speaking truthfully or lying. (He also argues that what I do at  $t_2$  is both good and bad, for parallel reasons, but we will not discuss those arguments separately.)

Finally, al-Taftāzānī assumes that

C. Goodness and badness are contradictories,

and so concludes that our attempt to apply reason to matters of good and bad has led to incoherence.

Al-Taftāzānī does not say anything in defense of (T), (F), (EBB), (EGG), or (C), and we will not make an attempt to defend these principles on his behalf, or assess the fairness of attributing such principles to the Mu‘tazila. But we will note that the argument, as we have reconstructed it, fails, because neither (EBB) nor (EGG) apply. What I do at  $t_1$ —saying “I will lie tomorrow”—does not entail that I lie tomorrow or that I do not lie tomorrow. For those entailments to hold, we either need to assume both that I say it *and*

that it is true, or that I say it *and* that it is false. But the truth or falsehood of what I say at  $t_1$  is not part of what I do at  $t_1$ .

Perhaps al-Taftāzānī has slightly different entailment principles in mind, namely,

EGBB. If the goodness of doing A entails doing B and doing B is bad, then doing A is bad.

EBGG. If the badness of doing A entails doing B and doing B is good, then doing A is good.

These principles seem about as plausible as (EBB) and (EGG), but their primary virtue is that they actually apply to the cases. Although what I do at  $t_1$  does not entail that I lie at  $t_2$ , the *goodness* of what I do at  $t_1$  does. When I said, ‘I will lie tomorrow,’ what I did was good because what I said was true, so the goodness of what I did entails that I do something bad—tell a lie—at  $t_2$ .

## 5. THE FALLACY IN ONE SENTENCE

These entailment principles are messy, and it is not obvious that they are true. Would not it be better to find an example that combined the goodness and badness in a single act, at a single time? This is precisely what al-Taftāzānī proceeds to do. The example he gives is the Liar Paradox:

If we consider a proposition (*qadiyya*) whose purport is to declare itself not to be true, then truth and falsehood are entailed in it, as when you say that the sentence (*kalām*) I now speak is not true, for its truth entails the non-being of its truth and vice versa.

His presentation suggests the influence of al-Ṭūsī, who was the first in the tradition to describe the paradox abstractly in this way, as a problem generated by self-reference:

Thus, the following paradox can be generated: The first declarative sentence, which is a declaration (*khabar*) about itself, namely that it is false, is either false or true. If it is true, then it must be false, because it declares itself to be false. If it is false, then it must be true, because if it is said falsely, then it will become true, which is absurd.<sup>18</sup>

But al-Taftāzānī's immediate goals are quite different from al-Ṭūsī's. At first, al-Taftāzānī does not seem especially interested in the Liar Paradox as a paradox about truth and falsehood, but in what follows from it about goodness and badness. Given (T) and (F), since what I have said is both true and false, the act of saying it was both good and bad. The point of introducing the Liar Paradox, for al-Taftāzānī, was to simplify the earlier argument, involving two separate sentences and dubious goodness and badness

---

<sup>18</sup> For discussion and translation of Ṭūsī on the Liar, including translation of this passage, along with references to the original sources, see Alwishah and Sanson, "Early Arabic Liar," 125–127.



entailment principles. And note that, if al-Taftāzānī is serious about this, he needs to accept the consequence that the sentence is both true and false, and so cannot dismiss it as a fallacy, as al-Ṭūsī does.

## 6. A TEMPORAL LIAR CYCLE

Before he stops to consider the Liar Paradox more carefully, al-Taftāzānī first extends the analogy between this case and the earlier case, in which I say, ‘I will lie tomorrow’, by describing a Liar Cycle, that is, a set of two or more sentences that taken separately, generate no paradox, but taken together generate a paradox:

For if one says that the sentence (*kalām*) I speak tomorrow is not true, or that nothing I speak tomorrow is extrinsically true, and then tomorrow his only statement is, ‘the sentence (*kalām*) I spoke yesterday is true,’ then the truth of either the ‘tomorrow’ sentence or the ‘yesterday’ sentence entails the non-being of the truth in both cases, and vice versa.

Let us name the two sentences,

s<sub>1</sub>: ‘What I say tomorrow is not true.’

s<sub>2</sub>: ‘What I said yesterday is true.’

Suppose  $s_1$  is true. Then it follows that  $s_2$  is not true, since  $s_2$  is what I say tomorrow. But if  $s_2$  is not true, then  $s_1$  is not true, since  $s_1$  is what I said yesterday. Likewise, suppose  $s_1$  is not true. Then it follows that  $s_2$  is true, from which it follows that  $s_1$  is true. So the two sentences together constitute what is known as a Liar Cycle or Deferred Liar.

Liar Cycles are interesting because they suggest that we cannot solve the Liar Paradox by a simple ban on direct self-reference. Al-Ṭūsī's solution involves just such a ban:

If a declarative sentence is the same as that-about-which-it-declares, then it cannot be conceived to be true and false. For agreement cannot be conceived except as between two things, and we cannot conceive them as opposed. For if one thing is affirmed, then nothing will be denied, and if one thing is denied, then nothing can be conceived to be affirmed.<sup>19</sup>

Al-Ṭūsī's claim is that, when a sentence is its own subject—when it makes a claim about itself—it is not the sort of thing that can be true or false, because truth requires an agreement between two distinct things and falsehood requires disagreement between two distinct things. And so, he says,

---

<sup>19</sup> Alwishah and Sanson, "Early Arabic Liar," 127.

It is clear that this fallacy arises as the result of a judgment that applies truth and falsity to something to which they in no way apply, and to apply them in any way is the misuse of a predicate.<sup>20</sup>

But neither sentence  $s_1$  nor sentence  $s_2$  declares something about itself. Each is distinct from the other, so they are two distinct things, and so, for all that al-Ṭūsī has said, they are the sorts of things that can be true or false.

Al-Taftāzānī does not mention al-Ṭūsī, and he does not present his Liar Cycle as a counterexample to any proposed solution to the Liar, so we do not know whether this was something he saw but did not mention, or something he did not see. But Liar Cycles would prove important to the later tradition, as al-Dawwānī uses them both to argue against al-Ṭūsī and to develop his own explanation of why such sentences are neither true nor false.<sup>21</sup>

---

<sup>20</sup> Ibid.

<sup>21</sup> Al-Dawwānī, *Nihāyat al-kalām fī ḥal shubhat kull kalāmī kādhib*, 129.

7. “THE LITTLEST BIT OF A LITTLE BIT”

Al-Taftāzānī now turns, in paragraph [5], to consider the Liar Paradox itself. He tells us that he has “found nothing that quenched [his] thirst,” and says that nothing has come to him, by way of a solution, but “the littlest bit of a little bit.”

The idea he presents mirrors an idea from al-Ṭūsī. Al-Ṭūsī distinguishes two roles that a sentence can play. A sentence, G, can either be used as a sentence, as when I say,

G. Grass is purple.

Or it can be the subject of a sentence, as when I say,

A. ‘Grass is purple’ is false.

Since (G) is false, there is a sense in which, in the sentence (A), truth and falsehood are gathered, but this does not involve any contradiction: the subject of (A) is false, but (A) itself is true. We only get a contradiction when truth and falsehood are gathered together in a single sentence in the same role.

In similar fashion, al-Taftāzānī distinguishes two roles that truth and falsehood can play:

In the same way that truth and falsehood are conditions for judgment (*ḥāllan li-l-ḥukm*)—i.e., affirmative and negative properties, required of all propositions—they can be a judgment (*ḥukman*)—i.e., what is judged [in the sense that it] is predicated of something indirectly, as when we say, this is true and that is false.

So, to consider his example,

B. The sky is beneath us.

(B) is false. This is a property (B) has, but it is not part of the content of (B). Supposing (B) expresses my judgment, my judgment predicates a certain location to the sky; it does not predicate truth or falsehood to anything. So, in this case, falsehood is a *condition for judgment*, but it is not the *judgment*—that is, it is not what is judged of the sky.

By contrast, in (C), falsehood is what is judged,

C. ‘The sky is beneath us’ is false.

But (C) is true. So, in this case, the *condition for judgment* is truth, but what is judged is falsehood.

Note that, just as we saw with (A), there is a sense in which truth and falsehood are conjoined in (C), but without contradiction. Al-Ṭūsī describes the case as one in which the sentence is true, but its subject is false. Al-Taftāzānī describes it as one in which what is judged is falsehood, but the condition for that judgment is truth. And, like al-Ṭūsī, he points out that this is not enough for a contradiction:

And they are not contradictory to each other unless we consider them as two conditions for one judgment, or two judgments upon one subject, as opposed to if we consider one of them as a condition for judgment and the other as a judgment.

How might one attempt to use this insight to solve the paradox?

To see this, it helps to first sketch out the argument that generates the paradox. Suppose our sentence is,

L. L is false.

(L) is either true or false. We generate the paradox by first supposing that it is true, and arguing from its truth to its falsehood, and then supposing that it is false, and arguing from its falsehood to its truth. There are various ways to present each argument, but here is a simple version of the argument from false to true:

1. 'L is false' is false.
2. For any S and P, if 'S is P' is false, then S is not P.
3. Therefore, L is not false.
4. Therefore, L is true.

With this sort of argument in mind, consider what al-Taftāzānī says,

If we suppose it [i.e., the proposition that belongs to this fallacy, 'the sentence I now speak is not true,'] to be false, then nothing beyond the truth of its contradiction is entailed, that is, our saying of this sentence that it is true, so then truth occurs as a judgment on this specific [proposition], not as a condition for its judgment; rather the condition for its judgment is falsehood, as we are supposing,

and truth is the affirmative property which is a condition for the judgment of the opposite [proposition], and a judgment for the original specific [proposition], and thus it is impossible to gather two conditions for one judgment or two judgments upon one subject.

It is not clear how to interpret the text here, but perhaps al-Taftāzānī is tell us that in (1)—our assumption that L is false—falsehood is the *condition for judgment* but not the *judgment*, while in (4)—the conclusion that L is true—truth is the *judgment* but not the *condition for judgment*. And so we do not have a contradiction, because we have not managed to gather truth and falsehood together in the same sentence in the same role.

If this were right, then, by carefully policing the distinction between truth and falsehood as conditions for judgment and truth and falsehood as judgments, one might hope to block both the argument from false to true and the argument from true to false, and so avoid the paradox.

#### 8. “THE WEAKNESS IN HIS SOLUTION IS APPARENT”

We have already seen that al-Dawwānī and al-Dashtakī think little of al-Taftāzānī’s proposal. Al-Taftāzānī’s seems to share their dim assessment, as he immediately goes on to say,

However, the correct judgment regarding this proposition is to give up on a solution and admit the incapability of [solving] this paradox.

Only Shams al-Dīn Muḥammad b. Aḥmad al-Khafīrī (d. 1550), writing half a century after al-Dawwānī and al-Dashtakī, attempts to explain what is wrong with the solution:

Since the weakness in [his] solution is apparent—it is necessary that the one and only subject of this proposition is itself, which entails the conjunction of truth and falsity in one proposition, which is impossible—al-Taftāzānī admits that he has departed from the right path.<sup>22</sup>

It is not immediately obvious how this objection applies, as al-Taftāzānī does not say that the Liar Sentence has multiple subjects.

Consider again (1) and (4) from the argument from false to true:

1. 'L is false' is false.
4. L is true.

Al-Taftāzānī insists that (1) expresses the fact that falsehood is a condition for judgment, while (4) judges L to be true.

---

<sup>22</sup> Al-Khafīrī, *Ḥayrat al-fuḍalā'*, in Qarāmalekī, *12 Treatises*, 276.



One might hope to read this distinction off of the structure of each sentence. A sentence like (1) is of the form,

‘A is B’ is C,

and so ascribes a property to a judgment, and so seems apt for expressing a condition for judgment. But (4) is of the form,

A is B,

which ascribes a property to an object, and so seems apt for expressing a judgment.

But this distinction collapses once we remember that L is an abbreviation for ‘L is false’, so that (1) can just as well be written as,

1\*. L is false,

and (4) can just as well be written as,

4\*. ‘L is false’ is true.

So it is hard to see how we are supposed to tell whether a given sentence expresses a condition for judgment or a judgment.

But perhaps we can find the materials for resisting this collapse in al-Taftāzānī’s puzzling remarks about explicit and hidden differences:

For what differentiates the subject (*al-marja* ') is either some explicit difference, as in our saying, “‘the sky is beneath us’ is true (or false),” or something hidden, as in the specific proposition that belongs to this fallacy.

The passage is difficult, but perhaps al-Taftāzānī means to distinguish metalinguistic subject terms that make the subject “explicit” from metalinguistic subject terms that “hide” the subject. So, in the sentence, “‘the sky beneath us’ is true”, the sentence that is the subject, ‘the sky beneath us’, is explicitly stated. But in the sentence ‘what I said yesterday is true’, the sentence that is the subject—whatever sentence it was that I said yesterday—is not explicitly stated, and so remains “hidden.”<sup>23</sup>

This distinction nicely captures an important difference between (1) and (1\*), and (4) and (4\*):

1. ‘L is false’ is false.
- 1\*. L is false.
4. L is true.
- 4\*. ‘L is false’ is true.

In (1) and (4\*), the subject is “explicit”; in (1\*) and (4), it is “hidden”.

---

<sup>23</sup> Thanks to an anonymous referee for suggesting this reading of the passage.

Armed with this distinction, one might insist on a syntactic test for the distinction between judgments and conditions for judgment: a sentence with an explicit sentential subject term expresses a condition for judgments; a sentence with subject term that hides the sentential subject expresses a judgment. But it is hard to see what to make of this claim when the underlying subject, whether made explicit or left hidden, is in fact the same sentence.

When a sentence has an explicit sentential subject term, as in “‘the sky is beneath us’ is true”, there will often also be an explicit difference between the inner predicate—‘is beneath us’—and the outer predicate, ‘is true.’ But this is not always true. When we express “the specific proposition that belongs to this fallacy” with an explicit sentential subject term, the inner predicate and the outer predicate appear to be the same:

1. ‘L is false’ is false.

What al-Taftāzānī needs, if he is going to block the argument that generates the paradox, is some way of positing a “hidden” difference between these two predicates.

We might try to make such a difference explicit by marking the distinction between judgments and conditions for judgment with subscripts. Recall that the argument from false to true begins with (1) and ends with (4), which together appear to show that the same sentence, L, is both false and true:

1. ‘L is false’ is false.

4. L is true.

And recall that al-Taftāzānī wants to insist that this is not a contradiction, because in (1), falsehood is a condition for judgment, while in (4), truth is what is judged. So, using subscripts to make his claim explicit, we can rewrite (1) and (4) as,

1\*\*. 'L is false<sub>judgment</sub>' is false<sub>condition for judgment</sub>.

4\*\*. L is true<sub>judgment</sub>.

The key premise of the argument from true to false, is premise (2):

2. For any S and P, if 'S is P' is false, then S is not P.

If al-Taftāzānī is right, and the argument begins with (1\*\*) and ends with (4\*\*), then that premise should be disambiguated as,

2\*\*. For any S and P, if 'S is P<sub>judgment</sub>' is false<sub>condition for judgment</sub>, then S is not P<sub>judgment</sub>.

From (1\*\*) and (2\*\*), we can infer,

3\*\*. L is not false<sub>judgment</sub>,

and from (3\*\*), given the assumption that truth<sub>judgment</sub> and falsehood<sub>judgment</sub> are contradictories, we can infer (4\*\*). But since truth<sub>judgment</sub> and falsehood<sub>condition for judgment</sub> are not contradictories, this result is not paradoxical.

The problem with this attempt to disarm the paradox is that (2\*\*) is not the only plausible disambiguation of (2). In particular, (2\*\*\*) looks equally plausible:

2\*\*\*. For any S and P, if ‘S is P<sub>judgment</sub>’ is false<sub>condition for judgment</sub>, then S is not P<sub>condition for judgment</sub>.

(2\*\*\*) says that, if, as a matter of external condition-for-judgment fact, it is false to judge that S is P, then, as a matter of external condition-for-judgment fact, S does not have P. And this seems correct: if, for example, as a matter of external fact, it is false to judge that the sky is beneath us, then, as a matter of external fact, the sky is not beneath us.

But once (2\*\*\*) is granted, we have what we need to argue from falsehood<sub>condition for judgment</sub> to truth<sub>condition for judgment</sub>. From (1\*\*) and (2\*\*\*), it follows that,

3\*\*\*. L is not false<sub>condition for judgment</sub>.

and so, assuming that falsehood<sub>condition for judgment</sub> and truth<sub>condition for judgment</sub> are contradictories, it follows that,

4\*\*\*. L is true<sub>condition for judgment</sub>.

And so we have the same sentence, L, that is both true<sub>condition for judgment</sub> and false<sub>condition for judgment</sub>, and al-Taftāzānī’s gambit fails.

So if al-Taftāzānī’s’ gambit is to succeed, he needs to give us some reason to accept principles like (2\*\*), but reject principles like (2\*\*\*). But this is not something he does, and it is not clear what such a reason could be.

9. ARE TRUTH AND FALSEHOOD *SHAR'ĪYĀN*?

Al-Taftāzānī's goal in this chapter is to show that goodness and badness are not intrinsic, by showing that attempts to use reason alone to judge goodness and badness lead to fallacies and contradictions. That, in turn, is meant to support the view that goodness and badness are *shar'iyān*—that is, grounded in divine command and knowable only by revelation. So, is the Liar Paradox merely a digression, with no real relevance to these broader goals, or is it meant to be of a piece with them?

On the one hand, al-Taftāzānī might think that truth and falsehood are knowable by reason, but goodness and badness are not. If so, the Liar is a digression, introduced as yet another way to show that reason is leads us to incoherence by dictating that truth-telling is always good and lying is always bad. And if so, it is a *problem* for al-Taftāzānī that Liar Paradox remains unsolved. Reason alone ought to be able to judge truth and falsehood without falling into incoherence, even if it cannot judge goodness and badness.

On the other hand, al-Taftāzānī might think that the Liar Paradox shows that truth and falsehood, like goodness and badness, are also not knowable by reason alone. On this reading, the Liar shows that the problems raised for goodness and badness are also problems for truth and falsehood. On this reading, it is *not* a problem for al-Taftāzānī that the Liar Paradox remains unsolved, because that is the point: reason alone cannot judge

truth and falsehood without falling into incoherence: “the correct judgment regarding this proposition is to give up on a solution and admit [our] incapability” of solving it.

Note that this reading of al-Taftāzānī’s final remarks on the incapability of solving the paradox crucially depends on the broader context of the argument that occurs in paragraphs [1] through [3]. So here it might matter that al-Dawwānī, al-Dashtakī, and al-Khafīrī focused only on paragraphs [4] through [9], and so read al-Taftāzānī’s remarks instead as an admission of failure.

To put the question bluntly: we know that al-Taftāzānī is a divine voluntarist, and that he thinks that goodness and badness are *shar‘iyān*. Might he go a step further, and embrace what we might call *alethic* voluntarism, the view that truth and falsehood are grounded in divine command, and so likewise *shar‘iyān*?

This seems like a radical view indeed, and we hesitate to attribute it to al-Taftāzānī. But recall that al-Taftāzānī’s *Sharḥ al-maqāṣid* is a commentary on al-Rāzī’s *al-Maṭalib al-‘āliyya*. As we have seen, al-Taftāzānī uses the Liar Paradox to show that the same speech act can be both good and bad, and so seems willing to argue that truth and falsehood are problematic in order to show that goodness and badness are not intrinsic. And in a similar spirit, al-Rāzī attacks truth and falsehood—arguing that no sentence is *ever* true or false—in order to show that truth and falsehood cannot ground the goodness and badness of telling the truth or lying.

Al-Rāzī begins with some definitions:

Truth is taken to be the agreement between a declarative sentence (*khābar*) and that about which it declares, and falsity, the non-agreement between a declarative sentence and that about which it declares. And it is well known that truth and falsity are two species under the genus, declarative sentence (*khābar*). And declarative sentence [is a species] under the genus of sentence (*kalām*), and a sentence is taken to be a vocalization (*lafẓ*) composed of a successive sequence of syllables.<sup>24</sup>

A vocalization (*lafẓ*) is a produced sound, but it need not have any syntax or semantics. The screech of a monkey, for example, is a *lafẓ*. A *kalām* is a *lafẓ* composed of successive syllables. Here we translate ‘*kalām*’ as ‘sentence,’ but it could also be translated as ‘speech.’ Note that by al-Rāzī’s definition, examples will include sub-sentential bits of speech, like an utterance of a *kalima* (word), like ‘lollipop’, or a phrase, like ‘the lollipop in the bag’, or a bit of nonsense, like ‘do doop dee do.’ A *khābar* (declarative sentence) is a *kalām* (sentence) that is true or false: so, not the utterance of a word or a phrase, or a string of nonsense, or a command or question, but the utterance of a complete declarative sentence, like ‘I want the lollipop in the bag.’

---

<sup>24</sup> Al-Rāzī, *al-Maṭālib*, part 3, 335.



Al-Rāzī proceeds to argue that each declarative sentence exists successively, one syllable after the next, so there is never a time at which a complete sentence exists:

And what exists of [a declarative sentence] is always nothing but a single syllable, and when [that syllable] is completed, the second syllable occurs, and so on, in this arrangement, until the last syllable of the word (*kalima*) occurs, and at this point the word (*kalima*) is complete. Based on this, the word (*kalima*) does not exist at all at any time or circumstance, and all that exists of it [at any time or circumstance] is a single syllable. And a single syllable is not a single sentence (*kalām*).<sup>25</sup>

Finally, he argues that, since complete sentences never exist, and truth and falsehood are properties of sentences, nothing is ever true or false. Furthermore, since sentences—that is, spoken sentences—are good and bad only if they are true or false, no sentence is ever good or bad:

If this is established, then we say: a single syllable is not a declarative sentence (*khābar*), and it is neither true nor false, and it is impossible for it to be what necessitates goodness or badness. With respect to the totality of a word (*kalima*), it does not exist at all, and that which does not exist at all cannot be the cause of

---

<sup>25</sup> Ibid.

something's being good or bad. By this proof, we establish that it is impossible for the sentence (*kalām*) to be good or bad, for it cannot be true or false.<sup>26</sup>

This is a bold and clever argument, to say the least. Whatever you think about its merits as an argument, note al-Rāzī's target. He wants to convince you that no sentence is ever true or false *because* he wants to convince you that no sentence is ever intrinsically good or bad. So, as with al-Taftāzānī, the detour into semantics is in service to the broader goal of showing that goodness and badness are not intrinsic. He says,

The difference is that you [Mu'tazila] say that being true necessitates the attribute of goodness, and being false necessitates the attribute of badness. We say that the truth is the totality, and this totality does not exist at all, and it is impossible for what does not exist to be a necessitator of an actual fixed attribute, and that claim [i.e., the claim that truth is a necessitator of an actual fixed attribute] is in contrast to what you say.<sup>27</sup>

But we do still use the words 'true' and 'false.' What are we getting at when we do so, if not an intrinsic attribute of sentences?

Instead, we say when we hear this sequence of syllables—which they agree make us aware of some meanings—without doubt we understand some of these

---

<sup>26</sup> Ibid, 335–336.

<sup>27</sup> Ibid, 336.

meanings, and at this point we form a belief about something. If the sentence (*kalām*) is false, then it becomes clear to us that our act is wrong (*bāṭil*). *And there is no other meaning of what is to be false except this.* And thus the difference [between us and the Mu‘tazila] becomes evident.<sup>28</sup>

So it appears that al-Rāzī’s wants to reverse the order of explanation: to say that a sentence is false is to say that it produces a belief in us that is *wrong*; to say it is true is to say that it produces a belief in us that is *right*.

But in what sense could a belief be right or wrong, if not in the sense that the belief is true or false? Remember that for al-Rāzī goodness and badness, and right and wrong, are *shar‘iyān*—grounded in divine command, and knowable not by reason, but only by revelation. So, putting two and two together, this suggests that for al-Rāzī, what it is good or bad *to believe* is likewise *shar‘iyān*, and that a belief, therefore, is true or false only insofar as it agrees or disagrees with divine command.

The text suggests, but does not force this reading. Perhaps al-Rāzī thinks that a right belief is one that corresponds to the facts, and a wrong belief is one that does not. On this reading, his primary concern is to reject truth and falsehood as properties of sentences—bits of language—and, instead treat them as properties of beliefs. But note that he does not say that a false sentence is one that produces a *false* belief. What he says is that a

---

<sup>28</sup> Ibid, 336, our emphasis.

false sentence is one that causes one to form a belief, where the *act* of forming that belief is *wrong*. And if he were to go on to say that the rightness and wrongness of belief is a matter of correspondence to the facts, then he would seem to be admitting that at least some rightness and wrongness is intrinsic rather than *shar‘iyān*, and we doubt this is something he would want to admit.

#### 10. “THE FALLACY OF THE ‘IRRATIONAL ROOT’”

We have nothing more to say about the substance al-Taftāzānī’s remarks on the Liar. In this last section of the paper, we turn to consider the puzzling name that he gives to the paradox. As we’ve mentioned, the later tradition follows al-Taftāzānī in calling the Liar Paradox the fallacy of the “irrational root.” It is not clear to us what lies behind this choice of name, or why the later tradition would have found it apt.

We have translated *‘jadr al-aṣamm’* as “irrational root.” It is a standard piece of mathematical terminology for a magnitude, like the square root of two, or the ratio of circumference to diameter, which cannot be expressed as a ratio of two numbers. But what does the Liar have in common with such magnitudes?

For the Greeks, the only *numbers* are the whole numbers. What we now think of as *rational numbers* they think of as magnitudes expressible as ratios of numbers. What we now think of as irrational numbers, they think of magnitudes not expressible as ratios of

numbers, and they think of those magnitudes primarily in geometric terms. So, for example, the square root of 2 is the ratio of the length of the side of a unit square to the length of its diagonal, and  $\pi$  is the ratio of the circumference of a circle to its diameter. For them, the problem posed by irrational magnitudes in general, and irrational roots in particular, was this: some quantities, essential to geometry, could not be expressed as ratios of numbers.<sup>29</sup>

Perhaps al-Taftāzānī's thought is this: when you begin doing geometry, and develop the concept of unit and then the concept of a ratio, it seems natural to suppose that all magnitudes resolve into a ratio of two numbers. But then you discover irrational roots, and realize that some magnitudes cannot be resolved in this way. So too, when you begin doing logic, and develop the concept of a declarative sentence, and the concept of truth and falsehood, it seems natural to suppose that all declarative sentences can be resolved into the true or the false. But then you discover the Liar, and realize that some sentences cannot be so resolved.

Al-Taftāzānī's own remarks provide little by way of explanation of what sort of analogy he had in mind:

---

<sup>29</sup> For an overview of the Greek concept of number in relation to irrational ratios, see Morris Kline, *Mathematical Thought from Ancient to Modern Times*. (Oxford: Oxford University Press, 1990), 32ff.

This is a fallacy that has perplexed the minds of the most intelligent and smartest people. For this reason I call it the fallacy of the irrational root (*jadhr al-aṣamm*).

‘Iṣmat Allāh b. Kamāl al-Dīn Maḥmmūd al-Bukhārī (16th c.), writing more than a century after al-Taftāzānī’s death, was equally puzzled. In his *al-Muntakhab min sharḥ risālat al-mughālaṭāt* (a Selection from Commentary on the Treatises of Fallacies), he offers up three possible explanations for the label, which trade on three different meanings of the word ‘*aṣamm*.’ The first meaning is the one from mathematics, and so his first explanation attempts to draw an analogy between the Liar Paradox and a corresponding problem about irrational roots in mathematics:

This is based on the inability of the intelligent people to solve it, just as they are unable to know the irrational root. And the root is the origin of a thing, and as he said in the *Muhadhdhab*,<sup>30</sup> “the root is the origin of arithmetic, and the irrational [root] of a number has no fraction, from the half to the tenth, as 11 and 13.” And it is reported in some of the books of arithmetic that no one knows the irrational

---

<sup>30</sup> Qarāmālekī (*12 Treatises*, 315 n. 272) suggests that this is a reference to Sajzī Maḥmūd bin ‘Umar’s *Muhadhdhab al-asmā’ fi martab al-ashyā’*.

root except God, and it is reported from some of the sages that, “[God] is to be praised, by saying, ‘praise He who knows the irrational root and no one else.’”<sup>31</sup>

So the idea is that the solution to the Liar Paradox is beyond the grasp of human knowledge, just as irrational roots are beyond the grasp of human knowledge. The report from the sages appears to be a reference to the famous mathematician, Muḥammad ibn Mūsā al-Khawārizmī (780–850), who says that we cannot apprehend the true nature of an irrational root, but God can:

An irrational root is that for which there is no way of knowing its truth (*ḥaqīqatahu*) through number, such as the root of two, root of three, or even root of ten, and it can be taken by an approximation, and its truth (*ḥaqīqatahu*) cannot be apprehended. And it is mentioned that one of the *Brāhima*’s praises in India was, “praise the one who knows the roots.”<sup>32</sup>

---

<sup>31</sup> Al-Bukhārī, *al-Muntakhab min sharḥ risālat al-mughālaṭāt*, in Qarāmālekī, *12 Treatises*, 315–316.

<sup>32</sup> Al-Khawārizmī, *Mafātīḥ al-‘ulūm*, 221. See also the marginal note in the Oxford manuscript of al-Khawārizmī’s *Algebra*, as reported and translated by Rosen:

Nobody can ascertain the exact truth of this, and find the real circumference, except the Omniscient: for the line is not straight so that its exact length might be found. This is called an approximation, in the same manner as it is said of the square-roots of irrational numbers that they are an approximation, and not the exact truth: for God alone knows what the exact root is (Frederic Rosen, *The Algebra of Mohammed Ben Musa* (London: The Oriental Translation Fund, 1831), 200).

Al-Bukhārī does not attempt to push the mathematical analogies any further than this epistemic idea, that the Liar and irrational roots are both unknowable by human reason. Or, perhaps, that the *truth* (*ṣidq*) of the Liar is unknowable by human reason, just as the *truth* (*ḥaqīqa*)—i.e., true essence or nature—of irrational roots is unknowable by human reason.

Are there any deeper analogies here, which might help explain al-Taftāzānī’s choice of label? Perhaps, but here we can only speculate.

One might try to press a structural analogy: just as rational roots involve a ratio between two numbers, you might think that truth and falsehood involve an agreement or disagreement between the judgment and the conditions for the judgment. And so just as irrational roots appear to be quantities that cannot be expressed as ratios between two numbers, the Liar Paradox might appear to give us a sentence that cannot stand in any such relation of agreement or disagreement.

If al-Taftāzānī had this sort of analogy in mind, it might help explain his own dissatisfaction with his proposed solution. You cannot “solve” the problem posed by  $\pi$  by insisting that every time we attempt to construct a ratio of circumference to diameter, we end up constructing some other rational ratio instead. So too you cannot “solve” the Liar



by insisting that every time we attempt to compare the judgment with the conditions for judgment, we instead end up making some other non-paradoxical comparison.<sup>33</sup>

Arabic mathematicians focused a great deal of energy on developing algorithms for calculating ever more precise rational approximations of irrational roots. Each approximation might seem to get us closer to the true nature of the irrational root, but, as a matter of fact, we never get any closer, since the true nature always recedes further away. Perhaps al-Taftāzānī saw an analogy here: it is not just that both are unknowable, but both are elusive. With the Liar, just when you think you've got it pinned down, it slips between your fingers and remains unresolved.<sup>34</sup>

So much, then, for speculative mathematical analogies. In addition to this explanation, al-Bukhārī offers two alternative non-mathematical suggestions for why al-Taftāzānī might have chosen to call this the fallacy of the “*jadhr al-aṣamm*.”

First, as al-Bukhārī points out, the word ‘*aṣamm*’ can also mean “solid” or “concrete”, or even “backbone,” understood as the principle that holds a solid thing together. So ‘*jadhr al-aṣamm*’ could mean something like “solid root”:

---

<sup>33</sup> Thanks to an anonymous referee for suggesting that we make explicit the possible relevance of this analogy to al-Taftāzānī's solution, and his attitude toward the solution.

<sup>34</sup> Thanks to Roshdi Rashed, who suggested a possibility along these lines in private correspondence.

It is possible that the naming of it is in virtue of ‘root’ (*jadhr*) being found in the language, in the meaning of *‘az bikh barkandan* (‘to pull up from the roots’). And, as was declared in *al-Ṣarāḥ*, *‘aṣamm*’ means backbone (*al-ṣulb*).<sup>35</sup> Thus, given that this fallacy invalidates the [principle] of the impossibility of conjunction of two contradictories, it is as if it cuts the origin of the thing, i.e., the backbone (*al-ṣulb*), which is the impossibility of the conjunction of two contradictories.<sup>36</sup>

So here the idea seems to be that the Liar Paradox threatens the principle of non-contradiction, and so threatens the “solid root” or “backbone” of logic itself. Whatever one might think about this speculation as an explanation of al-Taftāzānī’s use of the label, *‘jadhr al-aṣamm,*’ it suggests that by al-Bukhārī’s time there was a clear sense in the tradition that the Liar Paradox posed a serious threat to the foundations of logic.

Finally, *‘aṣamm*’ can mean “mute” or “deaf” (which is perhaps why the mathematicians chose it to translate the Greek word *‘alogos*’, which can mean ‘speechless’). Al-Bukhārī’s most entertaining speculation trades on this meaning of the word: perhaps the

---

<sup>35</sup> Qarāmalekī (*12 Treatises*, p. 316 n. 273) suggests that this is a reference to Qurashī Jamāl al-Dīn’s *al-Ṣarāḥ min al-ṣiḥāḥ*, a Persian-Arabic dictionary that would have been available al-Bukhārī.

<sup>36</sup> Al-Bukhārī, *al-Muntakhab*, in Qarāmalekī, *12 Treatises*, 316.

fallacy is so-called because the person who came up with it (and so the person who is its “root” or origin) was deaf:

It is possible that ‘root’ (*‘jadr’*) is a name attributed to the author of this fallacy, and he was deaf (*aṣamm*), unable to hear. But this can not be the case provided what we have reported of the scholar al-Taftāzānī, and [given] that it is reported that the author of this fallacy is Ibn Kammūna al-Baghdādī, who was one of the heretics (*malāḥida*).<sup>37</sup>

---

<sup>37</sup>Al-Bukhārī, *al-Muntakhab*, in Qarāmalekī, *12 Treatises*, 316. According to Pourjavady and Schmidtke, al-Bukhārī is among the earliest scholars to misattribute the paradox to Ibn Kammūna:

As a result of the popularity of the writings of Dawānī and Khafrī’s supercommentary on Ṭūsī’s *Tajrīd* among the philosophers of subsequent generation, Ibn Kammūna now became primarily known as a formulator of philosophical and logical sophistries (*shubhat*). [...] One of them was the liar paradox, known among Muslim philosophers as *shubhat jadhr al-aṣamm* or *shubhat kull kalāmī kādhib*. ‘Iṣmat Allāh b. Kamāl al-Dīn al-Bukhārī (10th/16th c.) was one of the earliest scholars to claim that this paradox originated with Ibn Kammūna. He even states that it is called *aṣamm* (“deaf”) because, so he maintained, Ibn Kammūna was deaf. (Pourjavady and Schmidtke, *A Jewish Philosopher of Baghdad*, 44.)

Note, however, that al-Bukhārī does not in fact endorse the claim that Ibn Kammūna was deaf, nor does he endorse this explanation of the meaning of *‘jadhr al-aṣamm*.’

## 11. CONCLUSION

Al-Taftāzānī's discussion of the Liar is both intrinsically interesting and historically important. He was the first to introduce Liar Cycles into the tradition, and the only figure in the tradition who seems to have connected the paradox with the broader debate between the Mu'tazila and Ashā'ira over the nature of goodness and badness. His own "littlest bit of a little bit" of a solution is suggestive even if it fails. And his decision to call the paradox the fallacy of the irrational root remains, for us, as for al-Bukhārī, an enigma.

*Illinois State University and Pitzer College*

## BIBLIOGRAPHY

Alwishah, Ahmed and David Sanson, "The Early Arabic Liar: The Liar Paradox in the Islamic World from the Mid-Ninth to the Mid-Thirteenth Centuries CE." *Vivarium*, 47 (2009), 97–127.

Al-Baghdādī, *Uṣūl al-dīn* (Dār al-Kutub al-'Ilmiyya: Beirut, 1981).

Al-Bukhārī, *al-Muntakhab min sharḥ risālat al-mughālaṭāt*. In Qarāmalekī, *12 Treatises*, 311–328.

Al-Dashtakī, *al-Muntakhab min ḥāshiyatihi ‘alā al-sharḥ al-jadīd lil-tajrīd*. In Qarāmalekī, *12 Treatises*, 3–14.

Al-Dawwānī, *al-Muntakhab min ḥāshiyatihi ‘alā al-sharḥ al-jadīd lil-tajrīd*. In Qarāmalekī, *12 Treatises*, 65–92.

Al-Dawwānī, *Nihāyat al-kalām fī ḥal shubhat kull kalāmī kādhib* (the Ultimate Saying in Solving the Paradox of ‘All what I say is False’), ed. A. Qarāmalekī, *Nāma Mofīd 5* (1996), Qom, University Mofīd.

Al-Ghazālī, *Just Balance*, trans. Richard McCarthy. Library of Classical Arabic Literature, under the General Editor Ilse Lichtenstadter, Volume IV (Boston: Twayne Publishers, 1980).

Al-Ghazālī, *Mi ‘yār al-‘ilm*, ed. S. Dunyā (Cairo: Dār al-Ma‘ārif, 1961).

Al-Ghazālī, *al-Qusṭās al-mustaqīm*, ed. V. Shalḥat (al-Maṭba‘a al-Kāthūlīkiyya, 1959).

Hourani, George. *Reason and Tradition in Islamic Ethics* (Cambridge: Cambridge University Press, 1985).

Al-Jabbār al-Qāḍī’s, *al-Mughnī*, VI, ed. A. al-Ahwānī (Cairo: al-Sharīka al-‘Arabiyya li-l-Ṭibā‘a wa al-Nashr, 1962).

Al-Khafīrī, *Ḥayrat al-fuḍalā’*. In Qarāmalekī, *12 Treatises on Liar*, 271–309

Al-Khawārizmī, *Maḡātīḥ al-‘ulūm*, ed. Ibrāhīm al-‘Abyyārī (Beirut: Dār al-Kitāb al-‘Arabī, 1989).

Kline, Morris. *Mathematical Thought from Ancient to Modern Times*. (Oxford: Oxford University Press, 1990).

Marmura, Michael. “Ghazali on Ethical Premises,” *Philosophical Forum* 1 (1969), 393-403.

Pourjavady, Reza and Sabine Schmidtke. *A Jewish Philosopher of Baghdad. ‘Izz al-Dawla Ibn Kammūna (d. 683/1284) and His Writings* (Leiden and Boston: E.J. Brill, 2006).

Qarāmalekī, Ahad. *12 Treatises on Liar Paradox in Shirāz School*, with the collaboration of Ṭaiiebe ‘Āref Nīyā (Tehran: Iranian Institute of Philosophy, 2007).

Al-Rāzī. *al-Maḡālib al-‘āliyya min al-‘ilm al-ilāhī*, III, ed. Aḡmad al-Ssaqā (Beirut: Dār al-Kitāb al-‘Arabī, 1987).

Rosen, Frederic. *The Algebra of Mohammed Ben Musa* (London: The Oriental Translation Fund, 1831).

Shihadeh, Ayman. “From al-Ghazali to al-Razi: 6th/12th Century Developments in Muslim Philosophical Theology.” *Arabic Sciences and Philosophy*, 15 (2005), 141–179.

Street, Tony. "Arabic and Islamic Philosophy of Language and Logic." In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Winter 2013.

Al-Taftāzānī, *Sharḥ al-Maqāṣid*, part IV, ed. 'Umara 'Abd al-Raḥmān, 2<sup>nd</sup> edition (Beirut: 'Ālam al-Kitāb, 1998).

Al-Taftāzānī, *Tahdhīb al-manṭiq wa al-kalām*. (Cairo: Maṭba'at al-Sa'āda, 1912).